

Understandable Proofs of Unsatisfiability

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Pythagorean Triples Problem (I) [Ronald Graham, early 80's]

Will any coloring of the positive integers with red and blue result in a monochromatic Pythagorean Triple $a^2 + b^2 = c^2$?

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Best **lower bound**: a bi-coloring of $[1, 7664]$ s.t. there is no monochromatic Pythagorean Triple [Cooper & Overstreet 2015].

Myers conjectures that the answer is **No** [PhD thesis, 2015].

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A bi-coloring of $[1, n]$ is encoded using Boolean variables x_i with $i \in \{1, 2, \dots, n\}$ such that $x_i = 1$ ($= 0$) means that i is colored red (blue). For each Pythagorean Triple $a^2 + b^2 = c^2$, two clauses are added: $(x_a \vee x_b \vee x_c)$ and $(\bar{x}_a \vee \bar{x}_b \vee \bar{x}_c)$.

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Theorem ([Heule, Kullmann, and Marek (2016)])

$[1, 7824]$ can be bi-colored s.t. there is no monochromatic Pythagorean Triple. This is impossible for $[1, 7825]$.

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4 CPU years computation, but 2 days on cluster (800 cores)

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200 terabytes proof, but validated with verified checker

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Posted by [BeauHD](#) on Monday May 30, 2016 @08:10PM from the red-pill-and-blue-pill dept.

THE CONVERSATION

Academic rigour, journalistic flair

76 comments



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200 Terabytes. Thats about 400 PS4s.

SPIEGEL ONLINE

Moshe's Question

“Will the size of the proof decrease if you enlarge the interval?”

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Transition point: $n = 7825$

- ▶ Initial: $\#cls = 18944$, $\#var = 6495$, $\rho = 2.92$
- ▶ Preprocessing: $\#cls = 14672$, $\#var = 3746$, $\rho = 3.92$
- ▶ Proof size: 200 terabyte

Moshe's Question

“Will the size of the proof decrease if you enlarge the interval?”

Transition point: $n = 7\,825$

- ▶ Initial: $\#cls = 18\,944$, $\#var = 6\,495$, $\rho = 2.92$
- ▶ Preprocessing: $\#cls = 14\,672$, $\#var = 3\,746$, $\rho = 3.92$
- ▶ Proof size: 200 terabyte

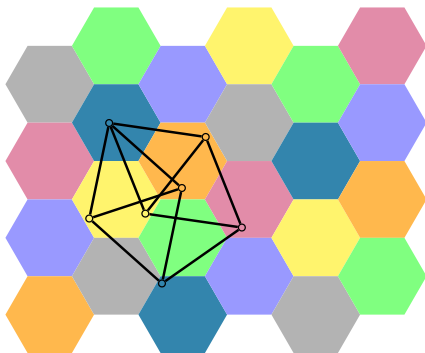
Way beyond transition point: $n = 1\,000\,000$

- ▶ Initial: $\#cls = 3\,961\,284$, $\#var = 866\,075$, $\rho = 4.57$
- ▶ Preprocessing: $\#cls = 635\,664$, $\#var = 64\,128$, $\rho = 9.91$
- ▶ Proof size: 1 terabyte

Chromatic Number of the Plane

The Hadwiger-Nelson problem:

How many colors are required to color the plane such that each pair of points that are exactly 1 apart are colored differently?



- ▶ The Moser Spindle graph shows the lower bound of 4
- ▶ A coloring of the plane showing the upper bound of 7

Small 5-Chromatic Unit Distance Graph

Use similar techniques:

- ▶ create a large, dense graph with chromatic number 5
- ▶ construct a small proof of unsatisfiability and extract core

